Electric polarizability of nuclei and a longitudinal sum rule

A.I. L'vov
P.N. Lebedev Physical Institute,
Leninsky Prospect 53, Moscow 117924, Russia

Abstract

Recently, a longitudinal sum rule for the electric polarizability of nuclei was used to revise a relativistic correction in a dipole sum rule for the polarizability. This revision is shown to be wrong because of neglecting an asymptotic contribution in the underlying dispersion relation. The status and correct use of the longitudinal sum rule is clarified.

I. INTRODUCTION

The electric $\bar{\alpha}$ and magnetic $\bar{\beta}$ polarizabilities of hadronic systems are currently a subject of many experimental and theoretical studies. The polarizabilities characterize a response of internal degrees of freedom of the system to external soft electromagnetic fields and can be measured in reactions with real or virtual photons [1].

It was established long ago [2] that

$$\bar{\alpha} = \alpha_0 + \alpha_{\rm rec}, \tag{1}$$

where

$$\alpha_0 = 2 \sum_{n \neq 0} \frac{|\langle n | D_z | 0 \rangle|^2}{E_n - E_0} \tag{2}$$

is a sum over excited states $|n\rangle$ of the system, with $\vec{D}$ being the operator of the electric dipole moment, and

$$\alpha_{\rm rec} = \frac{Z^2 e^2}{3Mc^2} r_E^2 \tag{3}$$

is a relativistic recoil correction of order $\mathcal{O}(c^{-2})$ which is determined by the mass M, the electric charge Ze, and the electric radius r_E of the system. We imply in Eq. (2) that a disconnected contribution of the electric polarizability of the vacuum is subtracted [3], so that α_0 is not necessarily positive. For the sake of clarity, we keep in formulas the speed of the light, c. We explicitly consider only the spinless case. When the system's spin $= 1/2$, a few minor modifications are necessary in (3) and in equations like (8), (11), or (18) below. They do not affect $\mathcal{O}(c^{-2})$ terms discussed in this paper. Since the term α_0 can be written as a weighted integral of the unretarded electric dipole cross section, we will refer to Eq. (1) as the dipole sum rule.

Recently, Eq. (1) has been questioned. Evaluating a longitudinal sum rule for the electric polarizability [4] in the case of a weakly-relativistic system (nucleus), Bernabeu *et al.* [5] found, apart from the terms α_0 and $\alpha_{\rm rec}$, an additional relativistic correction which is driven by a non-local (i.e. momentum-dependent or charge-exchange) part of the binding potential of the system and is of the same order $\mathcal{O}(c^{-2})$ as the recoil term $\alpha_{\rm rec}$.

The present paper is aimed to clarify the status of the longitudinal sum rule. We demonstrate that the sum rule used in the manner of Ref. [5] is inapplicable because of a violation of the underlying unsubtracted dispersion relation. When treated correctly, with an appropriate subtraction, the longitudinal sum rule leads to a result which is free from the potential-dependent correction and is consistent with the dipole sum rule.

II. DIPOLE AND LONGITUDINAL SUM RULES

We begin with reminding steps leading to both the dipole and longitudinal sum rules. Let us introduce the Compton tensor $T_{\mu\nu}$ which determines the amplitude of real or virtual photon scattering, $T = \epsilon'^*_\mu T^{\mu\nu}\epsilon_\nu$. In the following, we need only the 00-component of $T_{\mu\nu}$ found in the case of forward photon scattering on the system at rest,

$$T_{00}(\omega,q^2) = \sum_n \frac{\Big|\langle n,\vec q\,|\displaystyle\int \rho(\vec r)\exp(i\vec q\cdot\vec r)\,d^3r\,|0\rangle\Big|^2}{E_{n,q}-E_0-\omega-i0^+} + \begin{pmatrix}\omega\to-\omega\\ \vec q\to-\vec q\end{pmatrix}. \tag{4}$$

Here q is the 4-momentum of the photon and $\omega = cq_0$. Also, $\rho(\vec r)$ is the charge density, $|0\rangle$ is the initial state of the energy $E_0 = Mc^2$, and $|n,\vec q\rangle$ are intermediate states of energies $E_{n,q} = \sqrt{E_n^2 + c^2\vec q^{\,2}}$ and of the momentum $\vec q$. All the states are normalized as to have one particle in the whole volume $\Omega = \displaystyle\int d^3r$. We use the Gaussian units for electric charges, $e^2 \simeq \hbar c/137$. This removes $1/4\pi$ in equations like (2), (3), (8) or (11) below.

The low-energy behavior of T_{00} directly follows from Eq. (4). Separating the singular contribution of the unexcited state, $n=0$, and expanding the exponent in (4), one has

$$T_{00}(\omega,q^2) = T_{00}^{(n=0)}(\omega,q^2) + \vec q^{\,2}\alpha_0 + \mathcal{O}(\omega^2\vec q^{\,2},\ \vec q^{\,4}), \tag{5}$$

where α_0 is given by Eq. (2) with $\vec D = \displaystyle\int \vec r\,\rho(\vec r)\,d^3r$. The $(n=0)$-contribution is determined by the following covariant matrix element of the electromagnetic current $j^\mu = (c\rho,\vec j)$:

$$\langle 0,\vec q\,|\,j^\mu(0)\,|0\rangle = \frac{ceF(\tau)}{\sqrt{(2E_{0,q}\Omega)(2E_0\Omega)}}(E_{0,q}+E_0,\ c\vec q). \tag{6}$$

Here $F(\tau)$ is the electric form factor of the system which depends on the momentum transfer squared $\tau = -\vec q^{\,2} + (E_{0,q}-E_0)^2/c^2$ and is normalized as $F(0)=Z$. Accordingly,

$$T_{00}^{(n=0)}(\omega,q^2) = e^2F^2(\tau)\frac{(E_{0,q}+E_0)^2}{2E_{0,q}E_0}\,\frac{E_{0,q}-E_0}{(E_{0,q}-E_0)^2-\omega^2}. \tag{7}$$

When $q^2 \equiv q_0^2 - \vec q^{\,2} = 0$, Eqs. (5) and (7) give

$$T_{00}(\omega, 0) = -\frac{Z^2e^2}{Mc^2}\Big(1 - \frac{\omega^2}{3c^2}r_E^2\Big) + \frac{\omega^2}{c^2}\alpha_0 + \mathcal{O}(\omega^4), \tag{8}$$

where the electric radius of the system characterizes the slope of the form factor, $F'(0) = Zr_E^2/6$.

On the other hand, using an effective Lagrangian for a polarizable system [3], one can relate T_{00} with the commonly used polarizabilities $\bar{\alpha}$ and $\bar{\beta}$ which by definition determine a deviation of the low-energy Compton scattering amplitude from the so-called Born term,

$$T_{00}(\omega, q^2) = T_{00}^{\mathrm{Born}}(\omega, q^2) + \vec{q}^{\,2}\bar{\alpha} + \mathcal{O}(\omega^2\vec{q}^{\,2},\, \vec{q}^{\,4}). \tag{9}$$

The Born term describes photon scattering off a rigid (unpolarizable) charged "particle" of a finite size and is given by tree Feynman diagrams with propagators and electromagnetic vertices taken in the particle-on-shell regime. Up to the substitute $e \to eF(q^2)$, the Born amplitude coincides with the (virtual) Compton scattering amplitude in the scalar QED. In the case of forward scattering,

$$T_{00}^{\mathrm{Born}}(\omega, q^2) = -\vec{q}^{\,2}F^2(q^2)\frac{e^2}{Mc^2}\,\frac{4M^2c^2 - q^2}{4M^2\omega^2 - (q^2)^2}. \tag{10}$$

Therefore, Eqs. (9) and (10) give

$$T_{00}(\omega, 0) = -\frac{Z^2e^2}{Mc^2} + \frac{\omega^2}{c^2}\bar{\alpha} + \mathcal{O}(\omega^4). \tag{11}$$

Matching Eq. (11) with (8), one finally obtains [2] the dipole sum rule (1).

In essence, the dipole sum rule follows from Eq. (4) which, in turn, is valid when elementary constituents of the system have local couplings to the electromagnetic potential and, correspondingly, a seagull contribution S_{00} to the 00-component of the Compton tensor vanishes [6]. The same condition of the locality is required to suppress an exponential growth of the Compton scattering amplitude at high complex ω and to ensure the validity of dispersion relations.

The longitudinal sum rule just follows from matching Eq. (11) with a dispersion relation for $T_{00}(\omega, 0)$ which is known to be an even analytical function of ω. The imaginary part of T_{00} can be found from (4). Physically, it is related with the longitudinal cross section σ_L measured in electroproduction. Indeed, considering the amplitude of forward virtual photon scattering $T_L = \epsilon_L^{\mu\,*}T_{\mu\nu}\epsilon_L^\nu$ with $q^2 < 0$ and with the longitudinal polarization

$$\epsilon_L = \frac{1}{\sqrt{-q^2\vec{q}^{\,2}}}(\vec{q}^{\,2};\, q_0\vec{q}), \quad q \cdot \epsilon_L = 0, \tag{12}$$

and doing the gauge transformation $\tilde{\epsilon}_L^\mu = \epsilon_L^\mu - \lambda q^\mu$ with an appropriate λ, one can remove the space components of ϵ_L and arrive at the polarization vector

$$\tilde{\epsilon}_L = \sqrt{\frac{-q^2}{\vec{q}^{\,2}}}(1;\, \vec{0}). \tag{13}$$

Accordingly,

$$T_L(\omega, q^2) = \frac{-q^2}{\vec{q}^2} T_{00}(\omega, q^2). \tag{14}$$

The standard longitudinal cross section reads

$$\sigma_L(\omega, q^2) = \frac{4\pi}{q_{\rm eff}} \,\mathrm{Im}\, T_L(\omega, q^2), \quad q_{\rm eff} = q_0 + \frac{q^2}{2Mc}, \tag{15}$$

so that

$$\mathrm{Im}\, T_{00}(\omega, 0) = \frac{\omega^3}{4\pi c^3} \lim_{q^2\to 0^-} \frac{\sigma_L(\omega, q^2)}{-q^2}. \tag{16}$$

Both $\sigma_L(\omega, q^2)$ at $q^2 \le 0$ and $\mathrm{Im}\, T_{00}(\omega, 0)$ are manifestly positive.

Now, *assuming* an unsubtracted dispersion relation for $\omega^{-2} T_{00}$,

$$\omega^{-2}\, \mathrm{Re}\, T_{00}(\omega, 0) = -\frac{Z^2 e^2}{Mc^2\omega^2} + \frac{2}{\pi} \mathrm{P} \int_{\omega_{\rm th}}^\infty \mathrm{Im}\, T_{00}(\omega', 0) \frac{d\omega'}{\omega'(\omega'^2 - \omega^2)}, \tag{17}$$

one finally gets an unsubtracted longitudinal sum rule for $\bar\alpha$ which has been first suggested by Sucher and then rediscovered by Bernabeu and Tarrach [4]:

$$\begin{aligned}\bar\alpha &= \frac{2c^2}{\pi} \int_{\omega_{\rm th}}^\infty \mathrm{Im}\, T_{00}(\omega, 0) \frac{d\omega}{\omega^3} = \frac{1}{2\pi^2 c} \int_{\omega_{\rm th}}^\infty \lim_{q^2\to 0^-} \frac{\sigma_L(\omega, q^2)}{-q^2}\, d\omega \\ &= \sum_{n\ne 0} \frac{2c^2}{\omega_n^3} \Big(1 + \frac{\omega_n}{Mc^2}\Big) \Big| \Big\langle n,\, q_z = \frac{\omega_n}{c} \Big| \int \rho(\vec r) \exp(i\frac{\omega_n}{c} z)\, d^3 r \,\Big|0\Big\rangle \Big|^2, \\ &\qquad \omega_n = \frac{E_n^2 - E_0^2}{2E_0}.\end{aligned} \tag{18}$$

This sum rule should be used cautiously. Generally, the assumption that $\omega^{-2} T_{00}$ (or $T_L(\omega, q^2)$) vanishes at $\omega = \infty$ is not valid. Since the sum rule (18) necessarily gives a positive right-hand side, it is certainly violated whenever $\bar\alpha < 0$. Such a situation happens, for example, for polarizabilities of pions in the pure linear σ-model (i.e. in the σ-model without any heavy particles except for the σ) which are known to be negative. To one loop, $\bar\alpha_{\pi^\pm} = 4\bar\alpha_{\pi^0} = -e^2/(24\pi^2 m_\pi f_\pi^2) \simeq -2 \cdot 10^{-4}\,\mathrm{fm}^3$ [7]. Another instructive situation, where the sum rule (18) is violated, is QED, in which the amplitude T_L of γe scattering has a non-vanishing limit when $\omega \to \infty$ [8]. At last, in the hadron world, in which one can rely on the Regge model, the longitudinal amplitude T_L of forward scattering, as well as an amplitude with a transverse polarization, is expected to behave as $\omega^{\alpha_R(0)}$ with the Regge-trajectory intercept $\alpha_R(0) \simeq 1$. Therefore,

$$T_{00}(\omega, q^2) \sim \omega^{\alpha_R(0)+2} \quad \text{when } \omega \to \infty,\ q^2 = \text{fixed}, \tag{19}$$

and a subtraction in (17) is badly needed. Compared with a high-energy behavior of transverse components of the Compton tensor $T_{\mu\nu}$, the asymptotics (19) has an additional power of 2. This perfectly agrees with a presence in (4) of a time-component of the electromagnetic current j_μ which carries an additional Lorentz factor $\sim \omega$ in its matrix elements, cf. Eq. (6).

Since the unsubtracted dispersion relation for $\omega^{-2}T_{00}(\omega,0)$ is generally invalid, the integral in (18), even if convergent, does not necessarily coincide with $\bar{\alpha}$. Using in the dispersion relation a Cauchy loop of a finite size (a closed semi-circle of radius ω_M), we still can have the same dispersion integral, though truncated at $\omega=\omega_M$. But then the integral has to be supplemented with an asymptotic contribution which comes from the upper semi-circle (cf. Ref. [9]). Accordingly,

$$\bar{\alpha} = \frac{2c^2}{\pi}\int_{\omega_{\rm th}}^{\omega_M} \mathrm{Im}\, T_{00}(\omega,0)\frac{d\omega}{\omega^3} + \bar{\alpha}^{(\rm as)}, \tag{20}$$

where

$$\bar{\alpha}^{(\rm as)} = \frac{c^2}{\pi}\,\mathrm{Im}\int_{\omega=\omega_M\exp(i\phi),\ 0<\phi<\pi} T_{00}(\omega,0)\frac{d\omega}{\omega^3}. \tag{21}$$

In particular, if the cross section σ_L vanishes at high $\omega\sim\omega_M$ and $T_{00}(\omega,0)$ behaves as a *real* function

$$T_{00}(\omega,0) \simeq \cdots + \omega^{-2}A_{-2} + A_0 + \omega^2 A_2 + \omega^4 A_4 + \cdots, \tag{22}$$

the asymptotic contribution (21) is given by the coefficient A_2,

$$\bar{\alpha}^{(\rm as)} = c^2 A_2. \tag{23}$$

The subtracted form (20) of the longitudinal sum rule should be used instead of (18) when $\omega^{-2}T_{00} \not\to 0$ at high ω.

We conclude this section with the statement that both the dipole and longitudinal sum rules appear from the same quantity T_{00} and they must agree with each other. This is explicitly checked in the next section, in which Eq. (20) is evaluated to order $\mathcal{O}(c^{-2})$.

III. LONGITUDINAL SUM RULE FOR NUCLEI AND THE ASYMPTOTIC CONTRIBUTION

For a weakly-relativistic system like a nucleus, the sum I in (18) can be saturated with low-energy nonrelativistic excitations. Introducing a generic mass m of constituents and their velocity v, we have $\omega_n \simeq E_n - E_0 \sim mv^2$ and $\omega_n r \sim v \ll 1$ for essential intermediate states in Eq. (18). Accordingly, we can apply a v/c expansion of the relevant matrix elements,

$$\begin{aligned}\Big\langle n, q_z=\frac{\omega}{c}\Big|\int\rho(\vec r)\exp(\frac{i\omega}{c}z)\,d^3r\,\Big|0\Big\rangle \\ = eZ\delta_{n0} + \frac{i\omega}{c}\langle n|C_1(\omega)|0\rangle - \frac{\omega^2}{2c^2}\langle n|C_2(\omega)|0\rangle - \frac{i\omega^3}{6c^3}\langle n|C_3(\omega)|0\rangle + \cdots.\end{aligned} \tag{24}$$

Here $C_k(\omega)=\int(z-R_z(\omega))^k\rho(\vec r)\,d^3r$ for any integer k, and the diagonal-in-n operator $\vec R(\omega)$ relates intermediate states n at rest and at the momentum $q_z=\omega/c$. In terms of the boost generator $\vec K$ of the Poincare group which leaves the mass operator $\mathcal{M}c^2=\sqrt{H^2-c^2\vec P^2}$ of

the system invariant, $\vec{R}(\omega)$ is given by the equation $\vec{R}(\omega) = -\frac{c\vec{K}}{\omega}\mathrm{arcsh}\frac{\omega}{\mathcal{M}c^2}$. This equation is a paraphrase of $|n, q_z\rangle = \exp(-i\eta_n K_z)|n\rangle = \exp(iq_z R_z(\omega))|n\rangle$, where $\eta_n = \mathrm{arcsh}(\omega/E_n)$ is a rapidity. In the nonrelativistic limit, $\vec{R}(\omega)$ becomes an ordinary ω-independent operator of the center of mass. It is useful to notice that $\langle n|C_1(\omega)|0\rangle = \langle n|D_z|0\rangle$ when $n \neq 0$.

Using Eq. (24), expanding ω_n in generic powers of v/c, and keeping in Eq. (18) terms up to order $\mathcal{O}(v^2/c^2)$, we find that $I = I_0 + I_2$ where

$$I_0 = 2\sum_{n\neq 0} \frac{|\langle n|D_z|0\rangle|^2}{E_n - E_0}, \tag{25}$$

with the wave functions, energies, and the charge density including v^2/c^2 corrections, and [1]

$$\begin{aligned} I_2 &= 2\sum_{n\neq 0}\left[\frac{|\langle n|C_1|0\rangle|^2}{2Mc^2} + \frac{1}{c^2}(E_n - E_0)\left(\frac{1}{4}|\langle n|C_2|0\rangle|^2 - \frac{1}{3}\,\mathrm{Re}\left[\langle 0|C_1|n\rangle\,\langle n|C_3|0\rangle\right]\right)\right] \\ &= \Big\langle 0\Big|\,\frac{C_1^2}{Mc^2} + \frac{1}{4c^2}[[C_2, H], C_2] - \frac{1}{3c^2}[[C_1, H], C_3]\,\Big|0\Big\rangle. \end{aligned} \tag{26}$$

Terms of odd order in v/c vanish owing to the parity conservation. In Eq. (26) and in Eqs. (28), (30) below, we take all the quantities, including the Hamiltonian H, in the nonrelativistic approximation, when all C_k are ω-independent and hermitean.

In spite of an identical notation, the quantity I_0 is close but not identical to α_0, Eq. (2). The sum in I_0 involves intermediate states of only nonrelativistic energies $E_n - E_0 \sim mv^2$, whereas that in α_0 includes also antiparticle states with $E_n - E_0 \sim 2mc^2$. In the model of a relativistic particle moving in a binding potential, the difference between I_0 and α_0 is $\mathcal{O}(c^{-4})$ [10] and hence can be neglected in the present context.

Since the commutators in (26) explicitly depend on a binding potential V of the system, authors of Ref. [5] have concluded that $\bar{\alpha}$ explicitly depends on V too. This conclusion contradicts the dipole sum rule and technically is wrong because Eq. (20) contains an additional asymptotic contribution $c^2 A_2$ which cancels the V-dependent part of I_2.

To find A_2, we have to determine an asymptotic behavior of $T_{00}(\omega, 0)$ at energies beyond those which saturate the sums for I_0 and I_2. Accordingly, we consider energies ω which are high in comparison with a typical $E_n - E_0$ but still nonrelativistic (i.e. $\omega = \mathcal{O}(mv^2)$ and $qr = \mathcal{O}(v/c)$). This choice allows us to apply the v/c expansion (24). Using

$$\frac{1}{E_{n,q} - E_0 - \omega} + (\omega \to -\omega) \simeq -\frac{2}{\omega^2}(E_n - E_0) - \frac{1}{Mc^2} \tag{27}$$

and gathering in (4) all terms to order $\mathcal{O}(c^{-4})$, we indeed obtain a real asymptotics $T_{00}(\omega, 0) \simeq A_0 + \omega^2 A_2$ with the coefficients

[1] A similar equation of Ref. [5] contains also a few terms $\sim (E_n - E_0)^2/M$ which are of order $\mathcal{O}(c^{-4})$ and hence can be omitted.

$$
\begin{aligned}
A_0 &= -\frac{2}{c^2}\sum_{n\neq 0}(E_n-E_0)|\langle n|C_1|0\rangle|^2 - \frac{Z^2e^2}{Mc^2} + \mathcal{O}(c^{-4}),\\
A_2 &= 2\sum_{n\neq 0}\bigg[-\frac{|\langle n|C_1|0\rangle|^2}{2Mc^4} + \frac{1}{c^4}(E_n-E_0)\\
&\qquad\times\Big(-\frac{1}{4}|\langle n|C_2|0\rangle|^2 + \frac{1}{3}\,\mathrm{Re}\left[\langle 0|C_1|n\rangle\,\langle n|C_3|0\rangle\right]\Big)\bigg] + \frac{Ze}{Mc^4}\langle 0|C_2|0\rangle\\
&= -\frac{I_2}{c^2} + \frac{Z^2e^2}{3Mc^4}r_E^2.
\end{aligned}
\tag{28}
$$

A contribution of antiparticles to A_0 is absent. It is $\mathcal{O}(c^{-6})$ for A_2 and does not affect the leading term given in (28). When the system's Hamiltonian H and the charge density have the form

$$
H = \sum_{i=1}^{A}\Big(m_ic^2 + \frac{\vec{p}_i^{\,2}}{2m_i}\Big) + V, \quad \rho(\vec{r}) = \sum_{i=1}^{A} e_i\delta(\vec{r}-\vec{r}_i),
\tag{29}
$$

the asymptotic coefficients (28) are equal to

$$
\begin{aligned}
A_0 &= -\frac{1}{c^2}\langle 0|\,[[C_1,V],C_1]\,|0\rangle - \sum_{i=1}^{A}\frac{e_i^2}{m_ic^2},\\
A_2 &= -\frac{1}{4c^4}\langle 0|\,[[C_2,V],C_2]\,|0\rangle + \frac{1}{3c^4}\langle 0|\,[[C_1,V],C_3]\,|0\rangle.
\end{aligned}
\tag{30}
$$

The leading term A_0 in the asymptotics of T_{00} is determined by a sum of Thomson scattering amplitudes off the constituents of the system and by a V-dependent commutator which describes an enhancement in the Thomas-Reiche-Kuhn sum rule. This feature is very similar to that known for the amplitude of real Compton scattering [11]. Meanwhile the coefficient A_2 is entirely determined by V-dependent commutators and it vanishes when the binding potential V does not contain momentum-dependent or charge-exchange forces.

In the general case, $A_2 \neq 0$ and the electric polarizability $\bar{\alpha}$ found through the longitudinal sum rule (20) to order $\mathcal{O}(c^{-2})$ reads

$$
\bar{\alpha} = I_0 + I_2 + c^2A_2 = I_0 + \frac{Z^2e^2}{3Mc^2}r_E^2.
\tag{31}
$$

It does not contain potential-dependent commutators and agrees with the dipole sum rule. The quantities c^2A_2 and $-I_2$ are determined by the same matrix elements, and the only difference between them comes from the state $|n=0\rangle$ which is absent in the sum (26).

IV. CONCLUSIONS

We have shown that the longitudinal sum rule for $\bar{\alpha}$ in the unsubtracted form (18) used in Ref. [5] is not generally valid because of a divergence of the underlying unsubtracted dispersion relation (17). In particular, it is invalid for the case of photon scattering on nuclei in the next-to-leading order in $1/c^2$ because the binding potential V does not commute

with the charge density $\rho(\vec{r})$. Taking properly into account an asymptotic behavior of the longitudinal amplitude results in an additional asymptotic contribution $c^2 A_2$, Eq. (30), which removes artifacts predicted in Ref. [5] and brings the polarizability $\bar{\alpha}$ in accordance with the dipole sum rule (1).

The expansion in v/c, which was used in Section III to explicitly prove an agreement of the two sum rules, cannot be applied in case when real pion production off nuclei is taken into account since then $v/c \sim 1$. Still, a general analysis presented in Section II supports such an agreement, provided the subtracted form (20) of the longitudinal sum rule is used.

ACKNOWLEDGMENTS

Author appreciates useful discussions with V.A. Petrun'kin and A.M. Nathan. This work was supported in part by the Russian Foundation for Basic Research, grant 98-02-16534.

REFERENCES

[1] See, e.g., Proc. Workshop on Chiral Dynamics: Theory and Experiment (ChPT 97), Mainz 1997, eds. A. Bernstein, D. Drechsel, and Th. Walcher (Springer-Verlag, 1998), talks by D. Drechsel (nucl-th/9712013), M.A. Moinester and V. Steiner (hep-ex/9801008), B.R. Holstein (hep-ph/9710548), N. D'Hose, and others.

[2] V.A. Petrun'kin, Sov. Phys. JETP 13 (1961) 808; Nucl. Phys. 55 (1964) 197; V.M. Shekhter, Sov. J. Nucl. Phys. 7 (1968) 756.

[3] A.I. L'vov, Int. J. Mod. Phys. A 8 (1993) 5267.

[4] J. Sucher, Phys. Rev. D 6 (1972) 1798; J. Bernabeu and R. Tarrach, Phys. Lett. 55 B (1975) 183.

[5] J. Bernabeu, D.G. Dumm, and G. Orlandini, Nucl. Phys. A 634 (1998) 463.

[6] L.S. Brown, Phys. Rev. 150 (1966) 1338.

[7] A.I. L'vov, Sov. J. Nucl. Phys. 34 (1981) 289.

[8] E. Llanta and R. Tarrach, Phys. Lett. 78 B (1978) 586.

[9] V.A. Petrun'kin, Sov. J. Part. Nucl. 12 (1981) 278; A.I. L'vov, V.A. Petrun'kin, and M. Schumacher, Phys. Rev. C 55 (1997) 359.

[10] E.I. Golbraikh, A.I. L'vov, and V.A. Petrun'kin, Sov. J. Nucl. Phys. 37 (1983) 868.

[11] P. Christillin and M. Rosa-Clot, Nuovo Cim. 28 A (1975) 29.